INVESTIGATE RULES OF GAMES

Practice 2 – Suggested solution (Case 3)

Objective of the task:

To investigate and generate rules for the probabilities of all possible differences between the two numbers if you select two random cards (from 2 sets of 4 consecutive numbers).

Solution (Case 3: the 2 sets of cards are different, none of the cards are the same and we draw 2 cards from any of the 8 cards)
Level 1 -2:
I am going to use a table to find all possible outcomes when drawing two cards randomly (from the 2 sets consecutive numbers, says: 1,2,3,4,5,6,7,8, and the 2 sets of cards are mixed. That is after drawing the first card, there are still 7 cards to be drawn).
**(You might also use the tree diagram method to list the possibilities)

Possible 2nd card / 1st card	1	2	3	4	5	6	7	8
1	NA	(1,2) 1	(1,3) 2	(1,4) 3	(1,5) 4	(1,6) 5	(1,7) 6	(1,8) 7
2	(2,1) 1	NA	(2,3) 1	(2,4) 2	(2,5) 3	(2,6) 4	(2,7) 5	(2,8) 6
3	(3,1) 2	(3,2) 1	NA	(3,4) 1	(3,5) 2	(3,6) 3	(3,7) 4	(3,8) 5
4	(4,1) 3	(4,2) 2	(4,3) 1	NA	(4,5) 1	(4,6) 2	(4,7) 3	(4,8) 4
5	(5,1) 4	(5,2) 3	(5,3) 2	(5,4) 1	NA	(5,6) 1	(5,7) 2	(5,8) 3
6	(6,1) 5	(6,2) 4	(6,3) 3	(6,4) 2	(6,5) 1	NA	(6,7) 1	(6,8) 2
7	(7,1) 6	(7,2) 5	(7,3) 4	(7,4) 3	(7,5) 2	(7,6) 1	NA	(7,8) 1
8	(8,1) 7	(8,2) 6	(8,3) 5	(8,4) 4	(8,5) 3	(8,6) 2	(8,7) 1	NA
combination / difference								

I am going to do an experiment to find the experimental probabilities of each event.

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	Tally	Frequency	Experimental Probability $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
1	卌卌卌卌卌卌卌 |||	38	$\frac{38}{150} = 0.253$
2	卌卌卌卌卌卌	31	$\frac{31}{150} = 0.207$
3	卌卌卌卌卌 ||	26	$\frac{26}{150} = 0.173$
4	卌卌卌卌 |	21	$\frac{21}{150} = 0.140$
5	卌卌卌 |||	18	$\frac{18}{150} = 0.120$
6	卌卌 |	11	$\frac{11}{150} = 0.073$
7	卌	5	$\frac{4}{150} = 0.033$
	Total:	150	=1

I have drawn 2 cards randomly and find their differences for 150 times and recorded the above results.

Prediction-
According to the experimental probabilities, I predict that if I have done 300 times instead of 150, I will get around 76 times to be "1" as the two cards' difference.

Level 3-4:

Suggest how these patterns happen to other similar case-

If the numbers of the cards get bigger, says: 2,3,4,5,6,7,8,9

I predict that the probabilities of the corresponding events will be the same, as I can see from the following table, the final difference is still 1, 2, 3, 4, 5, 6, 7. The rules or patterns of the probabilities are similar to case 1 as shown above.

Possible 2nd card / 1st card	2	3	4	5	6	7	8	9
2	NA	(2,3) 1	(2,4) 2	(2,5) 3	(2,6) 4	(2,7) 5	(2,8) 6	(2,9) 7
3	(3,2) 1	NA	(3,4) 1	(3,5) 2	(3,6) 3	(3,7) 4	(3,8) 5	(3,9) 6
4	(4,2) 2	(4,3) 1	NA	(4,5) 1	(4,6) 2	(4,7) 3	(4,8) 4	(4,9) 5
5	(5,2) 3	(5,3) 2	(5,4) 1	NA	(5,6) 1	(5,7) 2	(5,8) 3	(5,9) 4
6	(6,2) 4	(6,3) 3	(6,4) 2	(6,5) 1	NA	(6,7) 1	(6,8) 2	(6,9) 3
7	(7,2) 5	(7,3) 4	(7,4) 3	(7,5) 2	(7,6) 1	NA	(7,8) 1	(7,9) 2
8	(8,2) 6	(8,3) 5	(8,4) 4	(8,5) 3	(8,6) 2	(8,7) 1	NA	(8,9) 1
9	(9,2) 7	(9,3) 6	(9,4) 5	(9,5) 4	(9,6) 3	(9,7) 2	(9,8) 1	NA
combination / difference								

Level 5-6:

I am going to find and explain the rules and patterns from the experimental results.

From the above experimental results and probabilities, I find out that:

(i) There are 56 possible combinations of getting 2 cards randomly from the 8 cards and there are 7 different events for the differences of the number of the two cards, which is 1, 2, 3, 4, 5, 6, 7 in my case.

(ii) There is the most chance to get a 1, P (two cards' difference is 1), which is 0.253 from my experiment.

(iii) Then the probabilities became smaller and smaller when the differences get bigger, until it is 7, which is 0.023, the smallest probabilities among all events according to my experimental result.

(iv) Getting P (two cards' difference is 1) will be 7 times of getting P (two cards' difference is 7)

My experimental results $= \frac{\text{P (two cards' difference is 1)}}{\text{P (two cards' difference is 7)}} = \frac{0.253}{0.033} = 7.6$ (which is kind of close to 7)

(v) The total of the experimental probabilities of all the outcomes is 1.

Test some of the pattern-

I can test one of the above rule is correct by finding its theoretical probabilities.

Since there are 56 difference outcomes and only 2 of them is "7"

therefore the theoretical probability P (two cards' difference is 7) $= \frac{2}{56} = \frac{1}{28} = 0.036$

which is quite close the experimental probabilities 0.033

Level 7-8
Test/Justify my rules/patterns are correct by finding all of the theoretical probabilities (giving more examples)

Difference of the two cards drawn (from 2 set of 4 consecutive numbers)	**Theoretical Probability** $=\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$	**Experimental Probability** $=\frac{\text{number of times the event occurred}}{\text{total number of trials}}$
1	$\frac{14}{56} = 0.250$	$\frac{38}{150} = 0.253$
2	$\frac{12}{56} = 0.214$	$\frac{31}{150} = 0.207$
3	$\frac{10}{56} = 0.179$	$\frac{26}{150} = 0.173$
4	$\frac{8}{56} = 0.143$	$\frac{21}{150} = 0.140$
5	$\frac{6}{56} = 0.107$	$\frac{18}{150} = 0.120$
6	$\frac{4}{56} = 0.071$	$\frac{11}{150} = 0.073$
7	$\frac{2}{56} = 0.036$	$\frac{4}{150} = 0.033$
Total:	1	=1

By comparing the experimental probabilities and the theoretical probabilities:

(i) There are 56 possible combinations of getting 2 cards randomly from the 8 cards and there are 7 different events for the differences of the number of the two cards, which is 1, 2, 3, 4, 5, 6, 7 in my case. (see table under level 1-2).
(ii) There is the most chance to get a 1, P (two cards' difference is 1), which is 0.250 (theoretically) and very close to my experimental probability (0.253).
(iii)Then the probabilities became smaller and smaller when the differences get bigger, until it is 7, which is 0.036 (theoretically), the smallest probabilities among all events.
(iv)Getting P (two cards' difference is 1) will be 7 times of getting P (two cards' difference is 7)

My experimental results $=\frac{\text{P (two cards' difference is 1)}}{\text{P (two cards' difference is 7)}} = \frac{\frac{14}{56}}{\frac{2}{56}} = 7$ (theoretically).

(v) The total of the experimental probabilities of all the outcomes is 1 (from the table).

Conclusions:
After going through the whole process of investigation and justification by finding the experimental and theoretical probabilities, I conclude that my rules and patterns for the probabilities of the difference by drawing 2 cards randomly from 2 sets of 4 consecutive cards are correct.

(i) There are 56 possible combinations of getting 2 cards randomly from the 8 cards and there are 7 different events for the differences of the number of the two cards, which is 1, 2, 3, 4, 5, 6, 7 in my case.
(ii) There is the most chance to get a 1, P (two cards' difference is 1),
(iii) Then the probabilities became smaller and smaller when the differences get bigger, until it is 7, the smallest probabilities among all events according to my experimental result.
(iv) Getting P (two cards' difference is 1) is 7 times of getting P (two cards' difference is 7)
(v) The total of the experimental probabilities of all the outcomes is 1.